Math 201 section **10.1**

Extra Formulas & Problems

Formula 1++: $\lim_{n\to\infty} \frac{\ln n}{n^c} = 0$ for any $c > 0$. Example: $\lim_{n\to\infty} \frac{\ln n}{n^{0.0001}} = 0$

Formula 7: $\lim_{n\to\infty} \frac{n!}{n^n} = 0$ (see Exercise 63)

Formula 8: $\lim_{n\to\infty} \sqrt[n]{n!} = \infty$ (Most students think the answer is 1 because $\lim_{n\to\infty} \sqrt[n]{n} = 1.$)

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1) Great problem: Find $\lim_{n\to\infty} (\frac{n+9}{n+2})^n$

Fastest Solution: $\lim_{n\to\infty} (\frac{n+9}{n+2})^n = \lim_{n\to\infty} \frac{(1+\frac{9}{n})^n}{(1+\frac{2}{n})^n} = \frac{e^9}{e^2} = e^7$

REMINDER : $1^\infty,\ 0^0,\ \infty^0$ are the undefined limits in exponents (why?) (Take their ln)

2) $\lim_{n\to\infty} (1+\frac{3}{n})^n = 1^\infty$!!!(undefined) The answer is ofcourse e^3 (by Formula 5).

3) $\lim_{n\to\infty} (\frac{n+9}{n+2})^n = 1^\infty$!!!(undefined) The answer here is e^7 (see Great problem 1 above)

4) $\lim_{n\to\infty} 1^n = 1$ because we are taking the limit of the sequence 1, 1, 1, 1, 1, …….

The following method in taking limits of exponents is acceptable to me **provided** we do NOT run into $1^\infty,\ 0^0,\ \infty^0$ (the Undefined limits in exponents). For example,

5) $\lim_{n\to\infty} (\frac{5n+1}{3n+2})^n = (\frac{5}{3})^\infty = \infty$

6) $\lim_{n\to\infty} (\frac{3n+9}{5n+2})^n = (\frac{3}{5})^\infty = 0$